

Variable Chaplygin Gas: Constraints from CMBR and SNe Ia

Geetanjali Sethi*, Sushil K. Singh, Pranav Kumar

*Department of Physics & Astrophysics,
University of Delhi, Delhi 110 007*

Deepak Jain

Deen Dayal Upadhyaya College, University of Delhi, Delhi 110 015

and

Abha Dev

Miranda House, University of Delhi, Delhi 110 007

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Abstract

We constrain the parameters of the variable Chaplygin gas model, using the location of peaks of the CMBR spectrum and SNe Ia “gold ” data set. Equation of state of the model is $P = -A(a)/\rho$, where $A(a) = A_0 a^{-n}$ is a positive function of the cosmological scale factor a , A_0 and n being constants. The variable Chaplygin gas interpolates from dust-dominated era to quintessence dominated era. The model is found to be compatible with current type Ia Supernovae data and location of first peak if the values of Ω_m and n lie in the interval $[0.017, 0.117]$ and $[-1.3, 2.6]$ respectively.

Keywords: cosmology:theory - chaplygin gas - dark energy - dark matter - CMBR
- SNeIa

1 Introduction

It is now well established that the expansion of the universe is accelerating. The direct evidence for acceleration comes from the Hubble diagram of Type Ia Supernovae (SN Ia) [1].

*getsethi@physics.du.ac.in

The observational results of SN Ia together with the anisotropy of cosmic microwave background radiation (CMBR) power spectrum and clustering estimates show that our universe is mainly made up of two components: dark matter and dark energy. The nature of dark matter and dark energy is not well understood. The dark matter contribute one-third of the total energy density of the universe. The dark energy, which is self interacting, unclustered fluid with large negative pressure, contributes roughly two-third of the total energy density of the universe(for latest review see [2]).

The simplest and the most favored candidate for the dark energy is the cosmological constant (Λ). Consequently, several models with a relic cosmological constant (Λ CDM) have been used to describe the observed universe. However most of them suffer from severe fine tuning problem [2, 3]. There are also models of dark energy with varying equation of state [4]. Another possibility for dark energy is quintessence which involve a slowly evolving and spatially homogeneous scalar field ϕ , [5, 6] or two coupled fields [7]. However , quintessence models also suffer from fine-tuning problem. This problem is usually highlighted as the “why now”, that is, why does the dark energy start dominating over the matter content of the Universe recently.

As an alternative to both the cosmological constant and quintessence, It is also possible to explain the acceleration of the universe by introducing a cosmic fluid component with an exotic equation of state, called Chaplygin gas [8, 9]. The attractive feature of such models are that they can explain both dark energy and dark matter with a single component. The equation of state for the Chaplygin gas is $P = -A/\rho$, where A is a positive constant. A more *generalized* model of Chaplygin gas is characterized by an equation of state

$$P_{ch} = -\frac{A}{\rho_{ch}^\alpha} \quad (1)$$

where α is a constant in the range $0 < \alpha \leq 1$ (the Chaplygin gas corresponds to $\alpha = 1$). By inserting eq.(1) into the energy conservation law we get the expression for the energy density as [10]

$$\rho_{ch} = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \quad (2)$$

where a is the scale factor and B is a constant of integration. As can be seen from the above equation, the Chaplygin gas behaves like a non relativistic matter at early times while at late times the equation of state is dominated by a cosmological constant $8\pi G A^{1/(1+\alpha)}$ which thus leads to the observed accelerated expansion. The Chaplygin gas models have been found to be consistent with SNeIa data [11], CMB peak locations [12] and other observational tests like gravitational lensing, cosmic age of old high redshift objects etc.[13] , as also with combination of some of them [14]. It has been shown that this model can be accommodated within the standard structure formation scenarios [9, 10, 15]. Therefore the Chaplygin gas model seems to be a good alternative to explain the accelerated expansion of the universe.However the Chaplygin gas model produces oscillations or exponential blowup of matter power spectrum that are inconsistent with observations[16].

Recently a *variable* Chaplygin gas model was proposed [17] and constrained using SNeIa

“gold” data [18]. In this letter, we try to constrain the variable Chaplygin gas model parameters using location of CMB peaks and SNeIa observations. We give the basic formalism of the model in section 2. Section 3 gives general formulae for CMB peaks. The best fit parameters have been calculated in section 4 by reduced χ^2 minimization of the distance modulus, followed by discussion and conclusions in section 5.

2 Variable Chaplygin gas model

We consider the proposed variable Chaplygin gas [17] characterized by the equation of state:

$$P_{ch} = -\frac{A(a)}{\rho_{ch}} \quad (3)$$

where $A(a) = A_0 a^{-n}$, is a positive function of the cosmological scale factor a . A_0 and n are constants. Using the energy conservation equation in flat FRW universe and eq.3, the variable Chaplygin gas density evolves as:

$$\rho_{ch}(a) = \sqrt{\frac{6}{6-n} \frac{A_0}{a^n} + \frac{B}{a^6}} \quad (4)$$

where B is a constant of integration.

The original Chaplygin gas scenario is restored for $n = 0$ and the gas behaves initially as dust-like matter and later as a cosmological constant. However, in the present case the Chaplygin gas evolves from dust dominated epoch to quintessence in present times (for details see [17] and references therein).

The Friedmann equation for a spatially flat universe reads as

$$H^2 = \frac{8\pi G}{3} \rho \quad (5)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. Therefore, the acceleration condition $\ddot{a} > 0$ is equivalent to

$$\left(3 - \frac{6}{6-n}\right) a^{6-n} > \frac{B}{A_0}, \quad (6)$$

which requires $n < 4$. This gives the present value of energy density of the variable Chaplygin gas

$$\rho_{ch0} = \sqrt{\frac{6}{6-n} A_0 + B} \quad (7)$$

where $a_0 = 1$. Defining

$$\Omega_m \equiv \frac{B}{6A_0/(6-n) + B}, \quad (8)$$

the energy density becomes

$$\rho_{ch}(a) = \rho_{ch0} \left[\frac{\Omega_m}{a^6} + \frac{(1 - \Omega_m)}{a^n} \right]^{1/2} \quad (9)$$

3 Location of CMBR peaks

The location of peaks are very sensitive to the variations in the parameters of the model and hence serve as a sensitive probe to constrain the cosmological parameters and discriminate among various models [19, 20]. The peak locations are set by the acoustic scale l_A , which can be interpreted as angle subtended by the sound horizon at the last scattering surface. This angle (say θ_A) is given by ratio of sound horizon to angular diameter distance, d_A of the last scattering surface:

$$\theta_A = \frac{a(t_{ls}) \int_0^{t_{ls}} c_s \frac{dt}{a(t)}}{d_A(t_{ls})} \quad (10)$$

where c_s is the speed of sound given in plasma by $c_s = 1/(3(1 + R))^{1/2}$ and $R = 3\rho_b/\rho_\gamma$ corresponds to the ratio of baryon to photon density. For the flat ($k = 0$) FRW model, the Acoustic scale $l_A = \pi/\theta_A$ is

$$l_A = \pi \frac{\tau_0 - \tau_{ls}}{c_s \tau_{ls}} \quad (11)$$

where τ_0 and τ_{ls} are conformal time today and at the last scattering surface.

For our calculations we assume: present value of scale factor $a_0 = 1$, the value of scale factor at the last scattering surface $a_{ls} = 1/1100$, $h = 0.71$, the density parameter for radiation and baryons at present $\Omega_{\gamma_0} = 9.89 \times 10^{-5}$, $\Omega_{b0} = 0.05$, average sound speed $\bar{c}_s = 0.52$ and spectral index for initial energy density perturbations, $n = 1$.

The location of the i^{th} peak in the spectrum is given by

$$l_i = l_A(i - \delta_i) \quad (12)$$

where the phase shift δ_i , caused by the plasma driving effect, is solely determined by the pre-recombination physics. We can approximate it with its value from standard cosmology [19]:

$$\delta_1 \approx 0.267 \left[\frac{r(z_{ls})}{0.3} \right]^{0.1} \quad (13)$$

where $r(z_{ls}) = \rho_r(z_{ls})/\rho_m(z_{ls}) = \Omega_{r0}(1 + z_{ls})/\Omega_{m0}$ is the ratio of radiation and matter density at the decoupling epoch. For the third peak,

$$\delta_3 \approx 0.35 \left[\frac{r(z_{ls})}{0.3} \right]^{0.1} \quad (14)$$

The Friedmann equation, using Eqn. 9 for a variable Chaplygin gas becomes

$$H^2 = \frac{8\pi G}{3} \left[\rho_{r0}(1+z)^4 + \rho_{b0}(1+z)^3 + \rho_{ch0} \left[\Omega_m(1+z)^6 + (1-\Omega_m)(1+z)^n \right]^{1/2} \right] \quad (15)$$

where ρ_{r0} and ρ_{b0} are the present values of energy densities of radiation and baryons respectively.

Using

$$\frac{\rho_{r0}}{\rho_{ch0}} = \frac{\Omega_{r0}}{\Omega_{ch0}} = \frac{\Omega_{r0}}{1 - \Omega_{r0} - \Omega_{b0}} \quad (16)$$

and

$$\frac{\rho_{b0}}{\rho_{ch0}} = \frac{\Omega_{b0}}{\Omega_{ch0}} = \frac{\Omega_{b0}}{1 - \Omega_{r0} - \Omega_{b0}} \quad (17)$$

Eqn. 15 becomes

$$H^2 = \Omega_{ch0} H_0^2 a^{-4} X^2(a) \quad (18)$$

where

$$X^2(a) = \frac{\Omega_{r0}}{1 - \Omega_{r0} - \Omega_{b0}} + \frac{\Omega_{b0}a}{1 - \Omega_{r0} - \Omega_{b0}} + a^4 \left(\frac{\Omega_m}{a^6} + \frac{1 - \Omega_m}{a^n} \right)^{1/2}. \quad (19)$$

Using the fact that $H^2 = a^{-4} \left(\frac{da}{d\tau} \right)^2$, we get

$$d\tau = \frac{da}{\Omega_{ch0}^{1/2} H_0 X(a)}, \quad (20)$$

so that Eqn.11 becomes

$$l_A = \frac{\pi}{c_s} \left[\int_0^1 \frac{da}{X(a)} \left(\int_0^{a_{ls}} \frac{da}{X(a)} \right)^{-1} - 1 \right]. \quad (21)$$

The ratio of radiation to matter density at the last scattering surface, r_{ls} in Eqn.13 becomes

$$r_{ls} = \frac{\Omega_{r0}}{a_{ls}^4 \left(1 - \Omega_{r0} - \Omega_{b0} \right) \left(\frac{\Omega_m}{a_{ls}^6} + \frac{(1-\Omega_m)}{a_{ls}^n} \right)^{1/2}}. \quad (22)$$

4 χ^2 minimization

We have used the SNe Ia data to constrain the parameters of the variable Chaplygin gas model. Here, to find out the luminosity distance we have taken into account the contributions from radiation and baryons together with the Chaplygin gas.

Using the Friedmann equation [15], in flat universe, the luminosity distance can be expressed as

$$d_L = \frac{c}{aH_0} \int_{a_{ls}}^1 \frac{da}{\Omega_{ch0}^{1/2} X(a)} \quad (23)$$

and the distance modulus

$$\mu_{th} = 5 \log \frac{H_0 d_L}{ch} + 42.38 \quad (24)$$

The best fit parameters are determined by minimizing [21]

$$\chi^2 = \sum_i \left[\frac{\mu_{th}^i - \mu_{obs}^i}{\sigma_i} \right]^2 - \frac{C_1}{C_2} \left(C_1 + \frac{2}{5} \ln 10 \right) - 2 \ln h \quad (25)$$

where

$$C_1 \equiv \sum_i \frac{[\mu_{th}^i - \mu_{obs}^i]}{\sigma_i^2} \quad (26)$$

$$C_2 \equiv \sum_i \frac{1}{\sigma_i^2} \quad (27)$$

We obtain the minimum $\chi^2 = 174.36$ for $\Omega_m = 0.22$ and $n = -2.8$ which are quite close to those obtained by [17]. The contour corresponding to 99% CL in figure 1 & 2 gives the range of $\Omega_m = [0.0, 0.36]$ and $n = [-41.3, 2.8]$ as tabulated in Table 1.

5 Discussion

The Variable Chaplygin gas behaves like non-relativistic matter during the early times and has the ability to drive the universe into an accelerated expansion in the recent times. From SNe Ia Gold data, this model is consistent with wide range of values for parameters Ω_m and n . The best fit of the model obtained by Guo & Zhang [17] gives $\Omega_m = 0.25$ and $n = -2.9$ with $\chi_{min}^2 = 173.88$. They performed the χ_{min}^2 test with the gold data set [18] using

$$H^2 = \frac{8\pi G}{3} \left[\rho_{ch0} \left[\Omega_m (1+z)^6 + (1 - \Omega_m) (1+z)^n \right]^{1/2} \right] \quad (28)$$

We repeated the test using the same SNe Ia Gold data set but took into account contribution from matter as well as radiation, as given in Eq. 15. The results obtained by us give $\chi_{min}^2 = 174.36$ at $\Omega_m = 0.22$ and $n = -2.8$. We find that matter and radiation do not contribute significantly to the expansion rate at redshifts for which the supernova data is available but we include them for completeness of our analysis. At 99% confidence level the χ_{min}^2 is obtained for values of parameters Ω_m and n lying in the range $[0.0, 0.36]$ and $[-41.3, 2.8]$ respectively.

The Figures 1 and 2 show the 1σ , 2σ and 3σ χ^2 contours of the SNe Ia Gold data and the location of CMBR peaks (using results of WMAP) at 1σ level in the Variable Chaplygin Gas model. In Figure 1, the contours 1 and 2 represent, for the first peak, the 1σ lower ($l_1 = 291.3$) and the upper ($l_1 = 220.9$) bounds on l obtained from WMAP data. In conclusion, the 1σ contours of the first peak of the WMAP combined with SNe Ia Gold sample restrains the parameter space to $\Omega_m = [0.017, 0.117]$ and $n = [-1.3, 2.6]$. In Figure 2, the contours 1 and 2 represent, for the third peak, the 1σ lower ($l_3 = 802.0$) and the upper ($l_3 = 816.0$) bounds on l obtained from WMAP data. The 1σ contours of the third peak of the WMAP combined with SNe Ia Gold sample restricts the parameter space to $\Omega_m = [0.018, 0.091]$ & $n = [-0.2, 2.8]$. The results have been tabulated in Table 1.

In Figures 3 and 4, we give the variation of the location of first and third peak respectively with n for different values of Ω_m . The horizontal lines in these two figures show the observational 1σ bounds for the two peaks. The bound on Ω_m from the first peak is $\Omega_m \leq 0.2$ and from the third peak is $\Omega_m \leq 0.18$.

Recently Guo and Zhang [22] have put constraints on the Variable Chaplygin Gas using X-Ray gas mass fractions in galaxy clusters and SNe Ia Gold data set. They have shown that the consistent range of Ω_m and n are $[0.043, 0.068]$ and $[1.18, 2.03]$ (1σ error bar) respectively. Our results based on location of WMAP 3^{rd} peak and SNe Ia at the 3σ level indicate the range $[0.018, 0.091]$ and $[-0.2, 2.8]$ for Ω_m and n respectively which accommodates the results obtained from X-Ray gas mass fractions in galaxy clusters and SNe Ia [22]. Though the original Chaplygin Gas is observationally ruled out, the Variable Chaplygin Gas can be further explored.

Table 1. *Bounds on Ω_m & n .*

Method	Reference	Ω_m	n
SNeIa(3σ)	This paper	$0.22^{+0.14}_{-0.22}$	$-2.8^{+5.6}_{-38.5}$
WMAP 1^{st} Peak + SNeIa(3σ)	This paper	$[0.017, 0.117]$	$[-1.3, 2.6]$
WMAP 3^{rd} Peak + SNeIa(3σ)	This paper	$[0.018, 0.091]$	$[-0.2, 2.8]$

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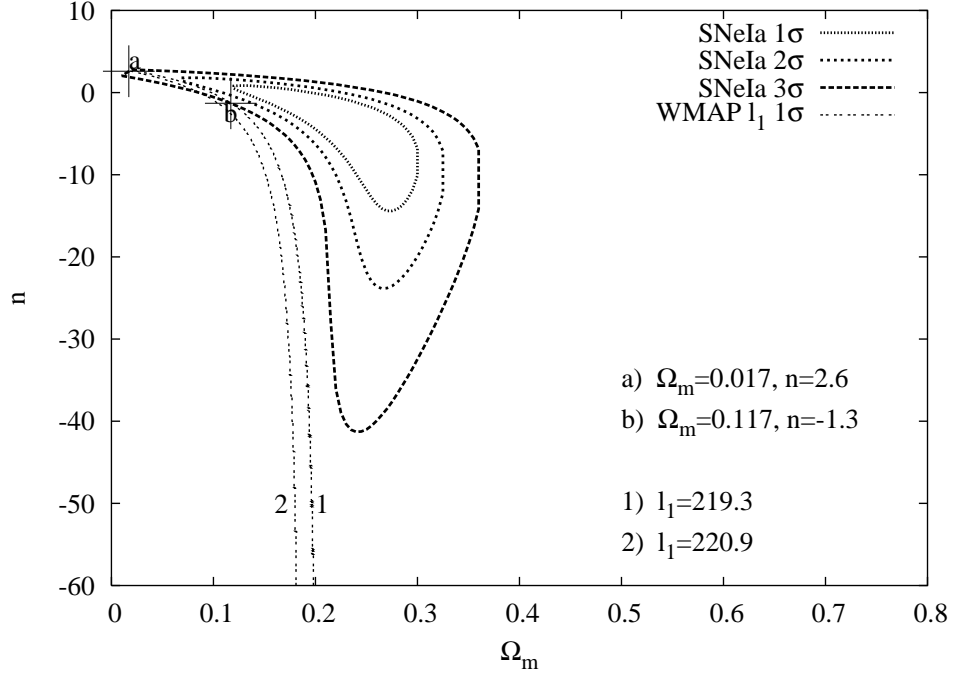


Figure 1: *The combined WMAP l_1 and SNeIa parameter space*

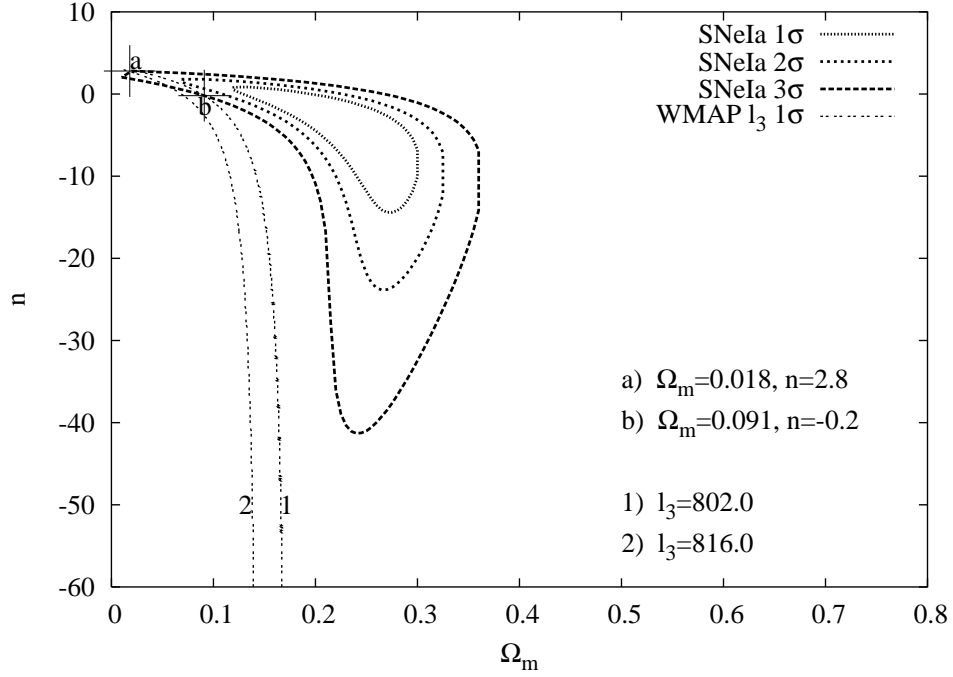


Figure 2: *The combined WMAP l_3 and SNeIa parameter space*

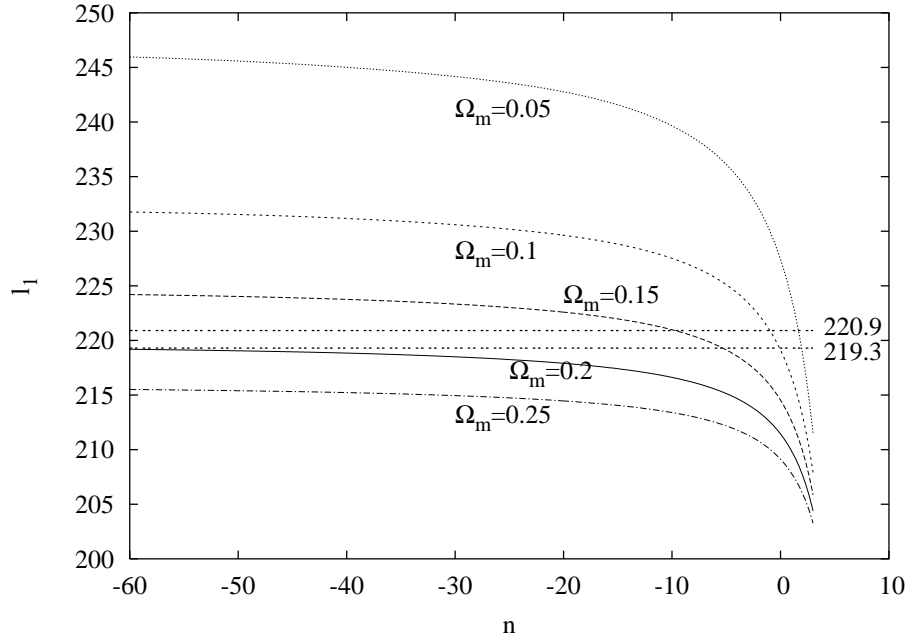


Figure 3: Variation of l_1 with n for different Ω_m . The horizontal lines correspond to the observational 1σ bound on l_1

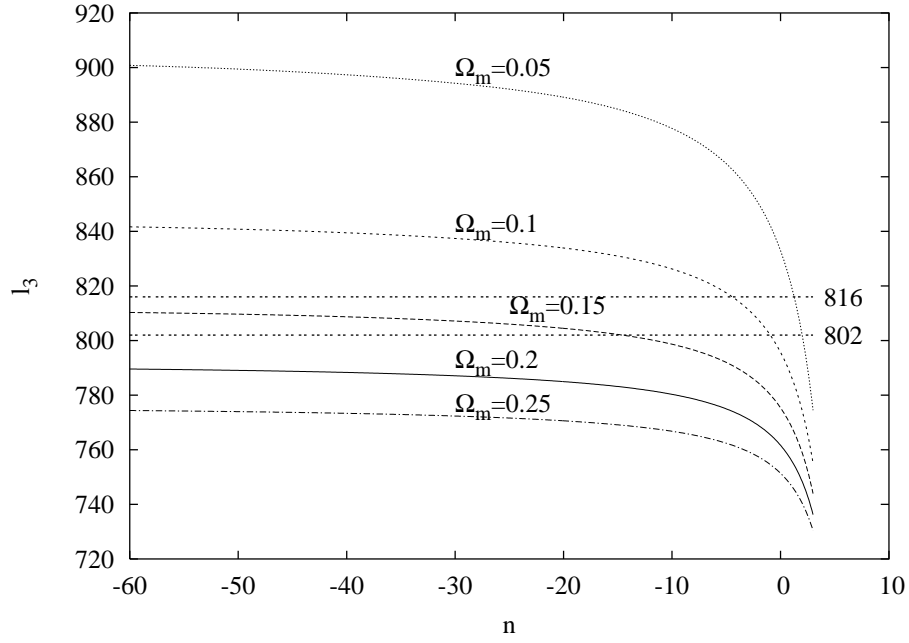


Figure 4: Variation of l_3 with n for different Ω_m . The horizontal lines correspond to the observational 1σ bound on l_3